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Assessment of Power Coefficient of an Offline Wind Turbine Generator System

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ABSTRACT In this paper, we present the design of an estimator for the assessment of the power coefficient of an offline wind turbine in a variable wind turbine generator system (WTGS) using a direct drive permanent magnet synchronous generator. It is shown that the estimator is supplying accurate estimates of the power coefficient. A further advantage of the estimator design presented is that it can be easily connected to WTGS where different types of generators and turbines are employed. The simulation results are presented using a graphic user interface and MATLAB Simulink.

KEYWORDS Wind Turbine Generator System (WTGS), Power Coefficient (C_p), Wind Speed Rising Rate (WSRR), Permanent Magnet Synchronous Generator (PMSG), MATLAB Simulink.

Introduction

Use of cost effective and reliable low carbon electricity generation source is becoming an important objective of energy policy in many countries [Béchir et al. 2012]. Over the past few years, wind energy indicated the fastest growth rate as compared to any other form of electricity generation [Jamdade et al. 2012a and 2012b]. The power coefficient (C_p) is an important parameter for the WTGS controller design. It represents the turbine efficiency in terms of fraction of energy extracted by the turbine from wind.

The power coefficient (C_p) is defined as the ratio of the power extracted by the wind turbine relative to the energy available in the wind stream. The Betz coefficient suggests that a wind turbine system can extract maximum 59.3 % of the energy in an undisturbed wind stream. The losses are attributed to different configurations of rotor blade profiles (blade surface roughness), finite wings, friction, and turbine designs (mechanical imperfections) in WTGS. Due to this, only 35 to 40% of the power available in the wind is extractable under practical conditions [Manyonge et al. 2012; Al-Bahadly, 2011]. The Betz Limit is an idealization and a design goal that designers try to reach in a real world of turbine. A C_p value of between 0.35 - 0.42 is a realistic design and goal for a working wind turbine [Bansal et al. 2002; Bansal et al. 2005].

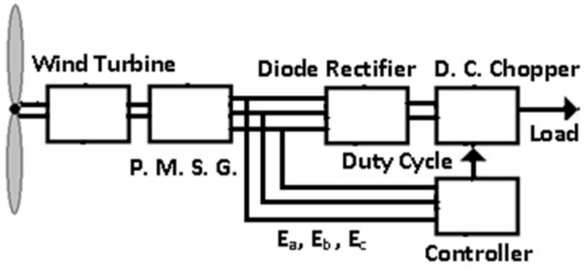

Figure 1 Block Diagram of WTGS Model.

For the extraction of maximum power from the rotor, the design of maximum power point controller is necessary which will track the C_p curve for the given rotor structure. So the assessment of power coefficient is a very important parameter of the wind generator controller system. Power coefficient (C_p) is a nonlinear function of mechanical power which depends on the pitch angle and the tip speed ratio (TSR). By considering the maximum pitch angle, TSR is needed to be calculated [Jangamshetti et al., 2001; Chang et al., 2007]. In this paper we have analyzed C_p value against the variations in TSR. We found that C_p getting maximum value for a particular value of the TSR. For getting C_p, the estimator is used and also for designing various control schemes. This estimator based model will have the advantage of being flexible and applicable to different turbines; thus making the controller as independent of the turbine parameters as possible as.

The paper is organized as follows. Section 2 describes the methodology for wind turbine generator system (WTGS) design used in the paper. In this section, mathematical models of components of WTGS are presented. Results and discussions are described in section 3 of this paper while conclusions are presented in section 4.

Mathematical Model of WTGS

WTGS consists of turbine, shaft and gear box, generator and power electronic converter to access the overall system behaviour. Block diagram of a WTGS model used in the study is shown in Fig. 1. The controller is used to assess C_p by which maximum power point tracking in WTGS is done. WTGS is modelled as follows: (i) the wind speed model, (ii) the wind turbine model, (iii) the drive train model, (iv) the PMSG model and (v) the power coefficient (C_p) estimator model.

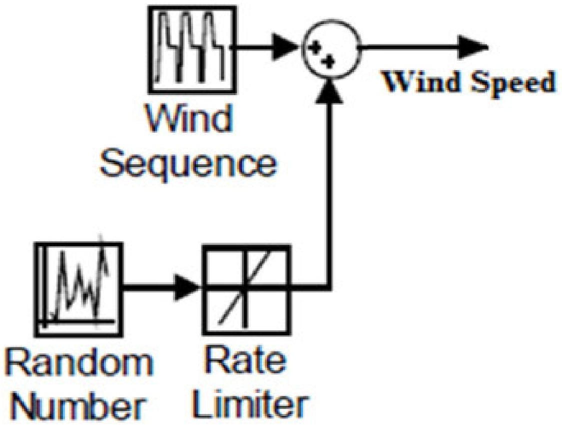

Figure 2 Wind Speed Model.

Wind Speed Model

A model is required that can properly simulate the spatial effects of wind behaviour, including gusting, rapid ramp changes and background noise. A variable wind speed model is obtained by adding a repetitive sequence of wind and random number blocks given in Eq. (1). Wind speed model is shown in Fig. 2.

$$V_w(t) = v_b(t) + v_r(t) + v_g(t) + v_n(t) \quad \text{Eq. (1)}$$

Rate limiter indicates the rate of change of wind speed. By putting rate limiter in the simulation we are ensuring that wind speed should not be above the cut out speed (furling speed) of the turbine and should not be under the cut in speed of the turbine. In both cases where wind speed is above the cut out speed and below the cut in speed, the turbine will not work properly.

Wind Turbine Model

The kinetic energy of the wind is given as:

$$KE_w = \frac{1}{2} mv^2 \qquad \text{Eq. (2)}$$

m is the air mass and is given as $m = \rho V_o = \rho \ A v$

Where v is the wind speed, ρ is the air density, V_o is the volume of air and A is the area covered by turbine blades.

By performing mathematical calculations wind power (P_w) is,

$$P_w = \frac{d}{dt}(KE_w) = \frac{1}{2} \rho A V^3 \qquad \text{Eq. (3)}$$

The mechanical power (P_m) extracted by the turbine from the wind is lesser to wind power (P_w). So, the power coefficient (C_p) of the turbine can be defined as,

$$C_p = \frac{P_m}{P_w}; \quad Cp < 1 \qquad \text{Eq. (4)}$$

Then the mechanical power (P_m) is given by,

$$P_m = \frac{1}{2} \rho \pi R^2 V^3 C_p \qquad \text{Eq. (5)}$$

Where, R is the radius of the rotor.

But C_p is a function of TSR (λ) of the wind turbine and the angle of the blade (β),

$$C_p = f(\lambda, \beta) \ \text{ with } \ \lambda = \frac{\omega R}{v} \qquad \text{Eq. (6)}$$

Where, ω is the angular rotation speed of the rotor.

The wind turbine torque on the shaft can be calculated in terms of mechanical power as:

$$T_m = \frac{P_m}{\omega} = \frac{1}{2} \rho \pi R^2 \frac{V^3}{\omega} C_p \qquad \text{Eq. (7)}$$

By introducing, $\lambda = \frac{\omega R}{v}$

$$T_m = \frac{1}{2} \rho \pi R^3 \frac{V^3}{\lambda} C_p \qquad \text{Eq. (8)}$$

The C_p can be obtained by data fields in the look-up tables or by approximating the coefficient using analytical function. The power coefficient (C_p) analytical function used to model the wind turbine is.

$$Cp(\lambda, \theta) = C_1 (C_2 \frac{1}{\beta} - C_3 \beta \theta - C_4 \theta^x - C_5) \ e^{-C_6 (1/\beta)} \qquad \text{Eq. (9)}$$

Since C_p function depends on the wind turbine rotor type, the coefficients C_1 to C_6 are $C_1 = 0.5$, $C_2 = 116$, $C_3 = 0.4$, $C_4 = 0$, $C_5 = 5$, $C_6 = 21$ and x is a constant value and can be different for different turbines.

Additionally, the parameter β is also defined as,

$$\frac{1}{\beta} = \frac{1}{\lambda + 0.08\theta} - \frac{0.035}{1 + \theta^3} \qquad \text{Eq. (10)}$$

Where, θ is the pitch angle and $\lambda = \frac{\omega R}{v}$

Drive Train Model

The drive train of a WTGS consists of elements like a blade pitching mechanism with a spinner, a hub with blades, a rotor shaft and a gearbox with breaker and generator. In this paper one mass drive train is considered with multi pole PMSG as shown in Fig. 3. At the time of modelling the damping and stiffness coefficients of the shaft are considered. The model of one mass drive train is implemented in MATLAB 7.10.0. The Math Works Inc., Natic, MA [2010].

$$\frac{d\omega}{dt} = \frac{T_m - T_e}{J} - \frac{B\omega}{J} \qquad \text{Eq. (11)}$$

B is the damping coefficient

ω is the angular velocity of the shaft

J is the moment of inertia. It is an equivalent mo-

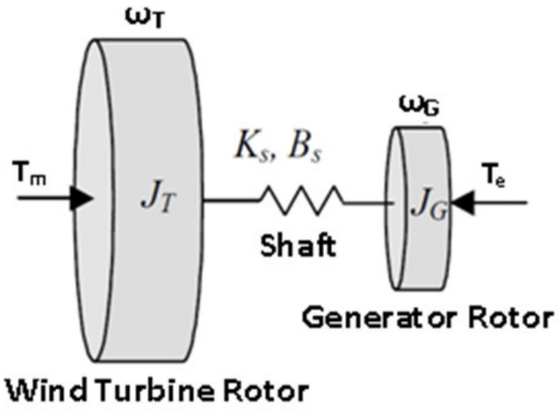

Figure 3 Drive Train Model.

ment of inertia of turbine and generator which is calculated from Eq. (11)

K_s is the stiffness factor which is a negligible so not considered in this work.

Permanent Magnet Synchronous Generator (PMSG) Model

Permanent Magnet Synchronous Generator (PMSG) provides an optimal solution for varying-speed wind turbines of gearless or single stage gear configuration. The output of the generator can be fed to the power grid directly so high overall efficiency can be achieved, while keeping the mechanical structure of the turbine simple.

Generated emf per phase is given in Eq. (12).

$$E = V_t + I_a\,(R_a + j\,X_s) \qquad \text{Eq. (12)}$$

The rotor reference frames of the voltages are obtained as Eq. (12) and Eq. (13).

$$\frac{d\,i_d}{dt} = \frac{1}{L_{ds} + L_{ls}}(-R_s\,i_d + \omega_e\,[L_{ds} + L_{ls}]\,i_q + v_d)$$

Eq. (13)

$$\frac{d\,i_q}{dt} = \frac{1}{L_{ds} - L_{ls}}(-R_s\,i_q + \omega_e\,[[L_{ds} + L_{ls}]\,i_d + \psi_f] + v_q)$$

Eq. (14)

The expression for the electromagnetic (EM) torque in the rotor is given by Eq. (15).

$$\tau_e = 1.5\,p\,([L_{ds} - L_{ls}]\,i_d\,i_q + i_q\,\psi_f) \qquad \text{Eq. (15)}$$

Torque developed by the turbine (T_t) and given to the generator as a mechanical torque (T_m) which is expressed as Eq. (16).

$$T_m = \frac{T_t}{G} \qquad \text{Eq. (16)}$$

Where G is the gear ratio.

Power Coefficient (C_p) Estimator Model

Our main objective is to design a power coefficient (C_p) estimator based on the WTGS model given in Eq. (17), Eq. (18) and Eq. (19). For this, we take C_p as a state variable instead of a parametric quantity and assume that the power coefficient is unknown and piecewise constant due that $\frac{dC_p}{dt} \cong 0$ for any time intervals (as Cp is constant value). As a result, system equations can be augmented by including the dynamics of C_p, yielding a system of order 3 given by:

$$\frac{d\,i_a}{d\,t} = \frac{y\,K_1\,I_f\,\omega}{L_a} - \frac{V_a}{L_a} - \frac{R_a\,I_a}{L_a} \qquad \text{Eq. (17)}$$

$$\frac{d\,\omega}{d\,t} = \frac{1}{2}\,\frac{Cp(\lambda)\rho\,\pi\,R^2\,V^3}{J\,\omega} - \frac{y\,K_1\,I_f\,I_a}{J} - \frac{B\,\omega}{J}$$

Eq. (18)

$$\frac{d\,C_p}{d\,t} \cong \frac{dC_p}{d\,\omega}\,\frac{d\,\omega}{d\,t} = 0 \qquad \text{Eq. (19)}$$

The estimation of C_p can be based on these three equations. However, in order to reduce computation complexity, we look for a reduced order model from which the estimation of C_p can be made. For this, we shall assume that the values of i_a, ω and u are measu-

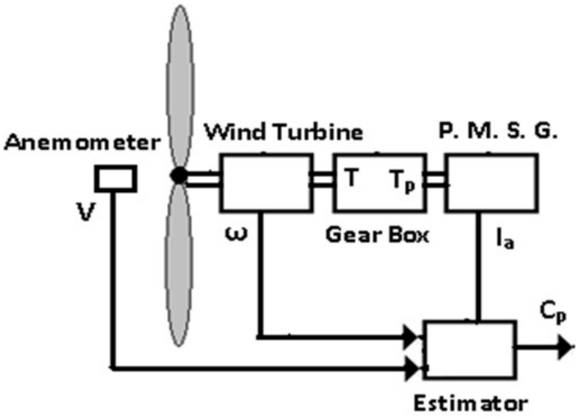

Figure 3 WTGS Estimator Model.

red quantities which means that they are accessible outputs of the WTGS. The mechanical power (P_m) and the wind speed (V) are inputs to the WTGS. Since i_a, V and ω are measured, they can be injected directly into a mechanical equation of the WTGS. As a result, only the mechanical dynamics of the WTGS can be considered for the estimation of C_p. So Eq. (18), Eq. (19) and Eq. (20) are reduced as.

$$\frac{d\omega}{dt} = \frac{1}{2}\frac{Cp(\lambda)\rho\ \pi\ R^2\ V^3}{J\omega} - \frac{y\ K_1 I_f\ I_a}{J} - \frac{B\omega}{J}$$

Eq. (20)

$$\frac{dC_p}{dt} \cong \frac{dC_p}{d\omega}\frac{d\omega}{dt} = 0$$

Eq. (21)

Turbine output fed to estimator $y = \omega$ Eq. (22)

Where, i_f, i_a and V are viewed as inputs to the system and ω as the output. This equation model (Eq. (20), Eq. (21)) is called as the second order reduced model and can be used for C_p estimation. Fig. 4 shows the WTGS estimator model.

Table 1 Variation in Cp and TSR with WSRR

WSRR	C_p	TSR
0	-0.69	16.75
0.04	-0.46	15.62
0.08	-0.26	14.48
0.12	-0.10	13.5
0.16	0.023	12.65
0.2	0.13	11.68
0.24	0.21	11.22
0.28	0.28	10.62
0.32	0.33	10.9
0.36	0.37	9.6
0.4	0.39	9.16
0.44	0.41	8.75
0.48	0.42	8.38
0.52	0.42	8.04
0.56	0.426	7.73
0.6	0.42	7.44
0.64	0.41	7.17
0.68	0.40	6.92
0.72	0.38	6.69
0.76	0.38	6.47
0.8	0.36	6.27
0.9	0.32	5.82
1	0.29	5.49

The wind speed profile is measured by anemometer and taken as a constant piecewise function in order to provide more realistic representation of power coefficient of practical system. If we keep rate limiters falling rate between 0 to -1, the values of wind speed fed to the turbine are much below the cut in speed of turbine so turbine is not going to operate. For those purpose rate limiters falling rate is kept as a constant and the value is -1. Variation in C_p and TSR with WSRR is shown in Table 1.

Results and Discussion

We evaluated the performance of the WTGS by carrying out simulation using MATLAB Simulink. The following numerical values of the model parameters are used for simulation studies:

Nominal power = P_m = 20 kW.
Cut in speed = 3 m/s.
Rated speed = 9 m/s.
Rotor radius = 2.3 m.
Pitch angle = $\beta = 0°$.
Stator resistance (R_s) = 1.5 ohms.
Stator inductances $L_d = L_q = 0.01$ mH.
Flux induced by magnets (ψ_f) = 0.1194 Wb.
Moment of inertia (J) = 2 Kg m^2.
Number of poles (p) = 4.

Chosen wind speed profile for MATLAB simulation is shown in Fig. 5. With this wind speed profile, variations in Cp and TSR values are observed. Variations in C_p with wind speed are shown in Fig. 6 for various sampled values of wind speed. Fig. 6, indicates that C_p variations with wind speed having WSRR below zero value giving us negative C_p which means that wind speed is below the cut in speed of the specified turbine generator. From the simulation, it clearly is observed that as WSRR increases C_p value increases but at a certain value of WSRR, C_p reaches to maximum value and drops down with increasing WSRR. The variation of C_p with wind speed shows that at a about 0.52 value of WSRR, C_p reaches to maximum value of 0.4272 while TSR has a value about 8.04. Table 1 shows that at zero WSRR, TSR is having maximum value with low C_p which is an undesirable result. But with increasing WSRR, TSR also reaches to a maximum value and again drops down. For a 20 kW wind turbine, the estimators provide C_p values with good convergence properties and faster response.

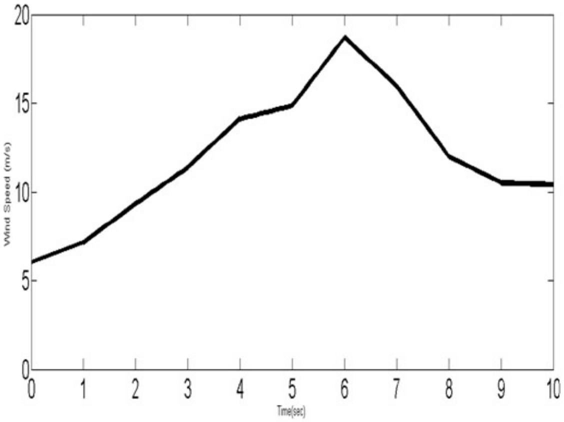

Figure 5 Wind Speed Profile.

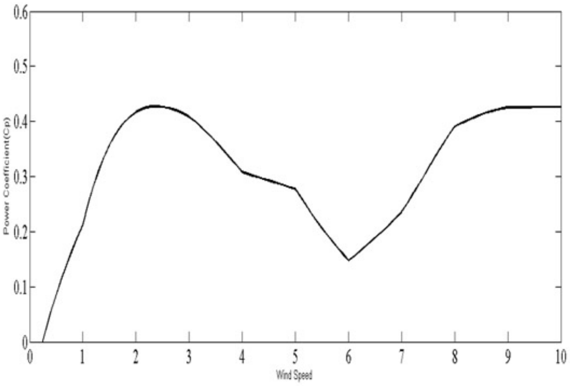

Figure 6 Estimated Power Coefficient (Cp) for given Wind Speed Profile.

Conclusion

In this paper, the C_p estimator is presented for calculation of power coefficient in a WTGS. The result shows that C_p estimator provides C_p maximum for good operation of a WTGS. For selected wind speed profile at about 0.52 value of WSRR, C_p reaches to maximum value of 0.4272 while TSR is having value about 8.04 where $\frac{dC_p}{dt} \cong 0$. The results obtained have verified the estimator's capability of giving good estimates of the power coefficients. This C_p estimator has many important advantages one of them is it allows fewer turbine parameter dependence of associated control system and it can easily be extended to any other WTGS where different generators and / or turbine types are used.

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